

Upravte výrazy s goniometrickými funkcemi (užijte vhodných vzorců):

a)  $\sin\left(\frac{\pi}{6} + x\right) + \sin\left(\frac{\pi}{6} - x\right)$  (cosx)

b)  $\cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{6} - x\right)$  (-sinx)

c)  $\sin\left(\frac{\pi}{4} + x\right) - \sin\left(\frac{\pi}{4} - x\right) + \cos\left(\frac{\pi}{4} - x\right) + \cos\left(\frac{\pi}{4} + x\right)$   
 $(\sqrt{2}(\sin x + \cos x))$

d)  $\cos \alpha + \cos(\alpha + 120^\circ) + \cos(\alpha + 240^\circ)$  (0)

e)  $\frac{\sin(u+v) - \sin(u-v)}{\sin(u+v) + \sin(u-v)}$   $\left(\frac{\operatorname{tg} v}{\operatorname{tg} u}\right)$

f)  $\frac{\cos(\pi+z)\sin\frac{3\pi}{2} + \cos\left(\frac{3\pi}{2}+z\right)\sin\frac{3\pi}{2}}{\sin\left(\frac{3\pi}{2}-z\right)\cos\pi - \sin\frac{3\pi}{2}\sin(\pi+z)}$  (1)

g)  $\sin\left(\frac{\pi}{4} + x\right)\cos x - \cos\left(\frac{\pi}{4} + x\right)\sin x$   $\left(\frac{\sqrt{2}}{2}\right)$

h)  $\sin(60^\circ + x)\sin(60^\circ - x) + \cos(60^\circ + x)\cos(60^\circ - x)$   $\left(\frac{\sqrt{2}}{2}\right)$   
 (cos 2x)

i)  $-\sin\left(\alpha - \frac{\pi}{6}\right)\cos\left(\alpha + \frac{\pi}{6}\right) + \cos\left(\alpha - \frac{\pi}{6}\right)\sin\left(\alpha + \frac{\pi}{6}\right)$   $\left(\frac{\sqrt{3}}{2}\right)$

j)  $-\frac{\sin u + \sin 2u}{\sin u - \sin 2u}$   $(\operatorname{tg} \frac{3}{2}u \cdot \operatorname{cot} g \frac{1}{2}u)$

k)  $\frac{\sin v + \sin 2v}{\cos v - \cos 2v}$   $\left(\operatorname{cot} g \frac{u}{2}\right)$

l)  $\frac{\sin t + \sin 3t}{\cos t + \cos 3t}$  (tg 2t)

m)  $\frac{\sin 5y - \sin 3y}{\cos 5y + \cos 3y}$  (tg y)

n)  $-\sin 218^\circ - \sin 202^\circ$  (cos 8°)

o)  $-\cos 477^\circ + \cos 927^\circ$   
 $(-\sqrt{2}\sin 18^\circ \text{ nebo } \sqrt{2}\sin 342^\circ)$

p)  $-\cos 154^\circ - \cos 746^\circ$  (0)

q)  $\sin \frac{5\pi}{12} - \cos \frac{9\pi}{5} - \sin \frac{\pi}{12} - \cos \frac{6\pi}{5}$

Řešte goniometrické rovnice s použitím vhodných vzorců

1) pomocí vzorců pro součet a rozdíl argumentů

a)  $\cos(x + 60^\circ)\cos(x - 60^\circ) = \frac{1}{4}$   $\{k\pi\}$

b)  $\sin\left(x + \frac{\pi}{4}\right)\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{4}$   $\left\{\frac{\pi}{6} + k\pi; \frac{5\pi}{6} + k\pi;\right\}$

2) pomocí vzorců pro součet a rozdíl goniometrických funkcí

a)  $\sin(x + 30^\circ) + \sin(x - 30^\circ) = \frac{\sqrt{3}}{2}$   $\left\{\frac{\pi}{6} + 2k\pi; \frac{5\pi}{6} + 2k\pi;\right\}$

b)  $\sin x + \sin\left(x + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$   $\left\{\pi + 2k\pi; \frac{5\pi}{3} + 2k\pi;\right\}$

c)  $\cos(x + 120^\circ) - \cos x = \sqrt{3}$   $\left\{\frac{7\pi}{6} + 2k\pi;\right\}$

d)  $\sin\left(\frac{\pi}{4} + x\right) - \sin\left(\frac{\pi}{4} - x\right) = \sin 2x$   $\left\{\frac{\pi}{4} + 2k\pi; \frac{7\pi}{4} + 2k\pi;\right\}$

## Další příklady:

1. Bez použití tabulek a kalkulačky určete:

a)  $\sin 75^\circ = \left(\frac{1}{4}(\sqrt{6} + \sqrt{2})\right)$       b)  $\cos 105^\circ = \left(\frac{1}{4}(\sqrt{2} - \sqrt{6})\right)$

c)  $\sin 15^\circ = \left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)$       d)  $\cos \frac{1}{12}\pi = \left(\frac{1}{4}(\sqrt{6} + \sqrt{2})\right)$

2. Zjednodušte:

a)  $\sin^2 x + \cos^2 x + \operatorname{tg}^2 x = \left(\frac{1}{\cos^2 x}\right)$       b)  $1 - \sin^2 x + \operatorname{cotg}^2 x \cdot \sin^2 x = (2\cos^2 x)$

c)  $\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x} = \left(\frac{2}{\cos^2 x}\right)$

3. Zjednodušte:

a)  $\cos\left(\frac{1}{6}\pi - x\right) - \cos\left(\frac{1}{6}\pi + x\right) = (\sin x)$       b)  $\cos 2x + \sin 2x \cdot \operatorname{tg} x = (1)$

c)  $\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = (\operatorname{tg} x)$

4. Řešte v R rovnice:

a)  $2\sqrt{3} \operatorname{cotg} \left(2x + \frac{1}{3}\pi\right) = -2 = \left\{\frac{\pi}{6} + k\frac{\pi}{2}\right\}$       b)  $\frac{5 + \sin x}{1 - \sin x} = 3 = \left\{\frac{7\pi}{6} + 2k\pi; \frac{11\pi}{6} + 2k\pi;\right\}$

c)  $\operatorname{cotg}^2 x = \sqrt{3} \operatorname{cotg} x = \left\{\frac{\pi}{2} + k\pi; \frac{\pi}{6} + k\pi;\right\}$       d)  $3 \cdot \operatorname{tg}^2 x = 1 = \left\{\frac{\pi}{6} + k\pi; \frac{5\pi}{6} + k\pi;\right\}$

e)  $4 \cdot \cos^2 x - 4 \cos x - 3 = 0 = \left\{\frac{2\pi}{3} + 2k\pi; \frac{4\pi}{3} + 2k\pi;\right\}$

f)  $(\cos x + 1) \cdot \cos x = 1 = \left\{\frac{\pi}{3} + 2k\pi; \frac{5\pi}{3} + 2k\pi; (2k+1)\pi\right\}$

g)  $\sin x + \frac{1}{\sin x} = 2 = \left\{\frac{\pi}{2} + 2k\pi\right\}$

5. Řešte v R rovnice:

a)  $3\sin^2 x = \cos^2 x = \left\{\frac{\pi}{6} + k\pi; \frac{5\pi}{6} + k\pi;\right\}$

b)  $3\sin^2 x + \cos x + \cos^2 x = 0 = \{(2k+1)\pi\}$       c)  $\operatorname{tg} x - 3 \operatorname{cotg} x = 0 = \left\{\frac{\pi}{3} + k\pi; \frac{2\pi}{3} + k\pi;\right\}$

d)  $\operatorname{tg} x - \operatorname{cotg} x - \frac{2}{\sqrt{3}} = 0 = \left\{\frac{\pi}{3} + k\pi; \frac{5\pi}{6} + k\pi;\right\}$       e)  $\sin^4 x - \cos^4 x = \frac{1}{2} = \left\{\frac{\pi}{3} + k\pi; \frac{2\pi}{3} + k\pi;\right\}$

f)  $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 1 - \cos 2x = \left\{\frac{\pi}{2} + k\pi\right\}$

g)  $\frac{1}{\sin^2 x} + \operatorname{cotg} x - 1 = 0 = \left\{\frac{\pi}{2} + k\pi; \frac{3\pi}{4} + k\pi;\right\}$

6. Řešte v  $\mathbb{R}$  rovnice:

a)  $\sin 2x + \cos x = 0$   $\left\{ \frac{\pi}{2} + k\pi; \frac{7\pi}{6} + 2k\pi; \frac{11}{6}\pi + 2k\pi \right\}$

b)  $\sin x - \cos 2x = 0$   $\left\{ \frac{3\pi}{2} + 2k\pi; \frac{\pi}{6} + 2k\pi; \frac{5\pi}{6} + 2k\pi \right\}$

c)  $\sin 2x = (\cos x - \sin x)^2$   $\left\{ \frac{\pi}{12} + k\pi; \frac{5\pi}{12} + k\pi; \right\}$

7. Řešte rovnice:

a)  $4 \sin\left(x - \frac{\pi}{2}\right) = 2$       b)  $8 \cos\left(2x - \frac{\pi}{8}\right) = 4\sqrt{3}$       c)  $\sqrt{2} \sin\left(\frac{x}{2} - \frac{\pi}{4}\right) = 1$

d)  $4\sqrt{3} \cos\left(\frac{x}{2} - \frac{\pi}{6}\right) = \sqrt{12}$       e)  $\operatorname{tg} \frac{x}{2} = 1$       f)  $6 \operatorname{cotg}\left(2x - \frac{\pi}{3}\right) = \sqrt{12}$

g)  $\sqrt{6} \operatorname{cotg}\left(\frac{x}{2} + \pi\right) = \sqrt{18}$       h)  $\operatorname{tg}\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{3}$

Výsledky:

a)  $\left\{ \frac{\pi}{2} + 2k\pi; \frac{7}{6}\pi + 2k\pi; k \in \mathbb{Z} \right\}$

b)  $\left\{ \frac{7}{48}\pi + k\pi; \frac{7}{24}\pi + k\pi; k \in \mathbb{Z} \right\}$

c)  $\{\pi + 4k\pi; 2\pi + 4k\pi; k \in \mathbb{Z}\}$

d)  $\left\{ \pi + 4k\pi; \frac{11}{3}\pi + 4k\pi; k \in \mathbb{Z} \right\}$

e)  $\left\{ \frac{\pi}{2} + 2k\pi; k \in \mathbb{Z} \right\}$

f)  $\left\{ \frac{\pi}{3} + k\frac{\pi}{2}; k \in \mathbb{Z} \right\}$

g)  $\left\{ -\frac{5}{3}\pi + 2k\pi; k \in \mathbb{Z} \right\}$

h)  $\left\{ \frac{\pi}{3} + k\frac{\pi}{2}; k \in \mathbb{Z} \right\}$

8. Zjednodušte:

a)  $\frac{1 + \operatorname{tg}^2 x}{1 + \operatorname{cotg}^2 x} = (\operatorname{tg}^2 x, x \neq k \cdot \frac{\pi}{2})$

b)  $\frac{\cos^2 x}{1 + \sin x} = (1 - \sin x, x \neq \frac{3}{2}\pi + 2k\pi)$

c)  $\frac{\sin x - \sin^3 x}{\cos x - \cos^3 x} = (\operatorname{cotg} x, x \neq k \cdot \frac{\pi}{2})$

d)  $1 - \sin^2 y + \operatorname{cotg}^2 y \cdot \sin^2 y = (2\cos^2 y, x \neq k \cdot \pi)$

e)  $\frac{\operatorname{tg} z}{1 + \operatorname{tg}^2 z} = (\sin z \cdot \cos z, x \neq (2k + 1) \cdot \frac{\pi}{2})$

9. Zjednodušte (využijte součtové vzorce):

a)  $\sin\left(\frac{\pi}{4} + x\right) - \sin\left(\frac{\pi}{4} - x\right) = (\sqrt{2} \sin x)$

b)  $\sin(u + v) \cdot \cos v - \cos(u + v) \cdot \sin v = (\sin u)$

c)  $\sin(20^\circ + y) \cdot \sin(20^\circ - y) + \cos(20^\circ + y) \cdot \cos(20^\circ - y) = (\cos^2 y - \sin^2 y)$